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THE USE OF VIBRATION CHARACTERISTIC TO UPDATE THE STRUCTURE MODEL

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ABSTRACT. Many authors studied algorithms adjusting the structure model based on modal data. This paper proposes an algorithm to detect the structure model using correlation factors between experimental and theoretical modal data in a damage library. The result from an experiment on 1-40 bridge (New Mexico USA) is presented to illustrate.

1. Introduction

Recently, the research of structure diagnosis using modal data has been developing rapidly. Many damage detection algorithms have been proposed to identify whether or not damage has occurred and to locate damage. However, in order to evaluate the load carrying capacity of a structure, its mathematical model need to be built correctly, relying on experimental data. The model will be complete if its modal data approximates the experimental modal data. The major problem in most identification algorithms is the incompleteness of the measured data: only a few points in structure are measured over a limited frequency range, but the finite element (FE) model of the structure contains a large number of degree of freedom. Therefore, updating a structure model relying on vibration data is difficult.

Detecting a model according to statistical technique is one of the algorithms used by many researchers, This technique was first suggested by Cawley and Adams [5]. By this technique, many possible damage scenarios within the finite element model are considered, and their effects on the predicted natural vibrations computed. The damage model is then identified as the one that seems to be closest to measured data. The two major problems with this technique are: 1) the time required to calculate a new set of natural vibration for every damage scenario and, 2) the algorithms to determine the actual damage correspond with one of the modeled scenarios. This paper develops an algorithm to detect the possible real models in the damage library.

2. Model updating overview

Although the range of model updating algorithms is large, the basic concepts are similar. A given structure can be modeled analytically and predictions of the response of the system can be made. Its response may also be measured and compared to the theoretical predictions. If results of the theoretical analysis and the measurements are different then some parameters of the theoretical model should be changed to reflect the characteristics of the physical structure. Assuming that the underlying structure of the model is satisfactory then parameters can be accepted. This is known as model updating.

FE analysis and experimental modal analysis (EMA) are two basic constituents of model updating algorithms for mechanical structures. FE analysis is a standard technique for modeling the dynamics of mechanical structures. In this technique the structure is split into regions of simple geometry (called elements) which intersect at points (called nodes). The equation systems of motion can be written as follows

$$\mathbf{M}\,\ddot{\mathbf{q}} + \mathbf{K}\,\mathbf{q} = \mathbf{f}, \tag{2.1}$$

where

$\mathbf{M}$, $\mathbf{K}$ are the mass and stiffness matrices,

$\mathbf{q}$ is the vector of generalized coordinates, that is the displacement at the nodes of structure,

$\mathbf{f}$ is the force applied to the structure at the nodes,

and the dot denotes differentiation with respect to time.

The natural frequencies and mode shapes are obtained by solving the following eigenproblem:

$$[-\omega_i^2 \cdot \mathbf{M} + \mathbf{K}]\boldsymbol{\phi}_i = 0, \tag{2.2}$$

here ω_i is i^{th} natural frequency and $\boldsymbol{\phi}_i$ is the corresponding mode shape.

Damping has not been mentioned in equation system (2.1). In general, damping is difficult to incorporate into a finite element analysis and furthermore its value is unknown.

In dynamic testing, the structure is usually excited by harmonic force or impulse force and its responses are measured. Using the Fast Fourier Transform for input and output signals, the transfer functions can be determined, then experimental modal data (natural frequencies, damping ratios, mode shapes) can be calculated by analyzing the transfer function [2, 4]. In this paper we assume modal data is given and do not discuss the measured data analysis to get the experimental vibration characteristics of the structure.

3. Model updating technique

Let's assume that the library of the theoretical modal data correspondings to the damage scenarios are given. For each damage type, we have to use the appropriate element model for calculating. For example, the physical or geometrical properties can be changed to simulate the damage. Because the changes of frequency are less sensitive to damage [2, 5], especially the minor damage, in this case the changes in mode shapes will be used to update the structure model. When the structure is damaged, mode shapes before and after damage are different. Therefore the appropriated model in the damage library may be detected using correlative comparison of measured and theoretical mode shapes.

Denote $x_i(j)$ $(i = 1, m)$ is i^{th} mode shape measured at point j $(j = 1, n)$, here n is the number of measured points, m-number of measured natural modes. Denote $y_{k\ell}(j)$ $(\ell = 1, m)$ is ℓ^{th} mode shape measured at point j corresponding k^{th} damage scenario.

The correlation factor between x_i and $y_{k\ell}$ can be written as follows:

$$R(x_i, y_{k\ell}) = \frac{\left(\sum\limits_{j=1}^{n} x_i(j) \cdot y_{k\ell}(j)\right)^2}{\sum\limits_{j=1}^{n} x_i^2(j) \cdot \sum\limits_{j=1}^{n} y_{k\ell}^2(j)} \tag{3.1}$$

with $i = 1, \dots, m$; $\ell = 1, \dots, m$; $k = 1, \dots, p$ (p is the number of theoretical damage scenarios).

If the k^{th} theoretical damage model is an appropriate model of the structure then:

$$R(x_i, y_{k\ell}) = \begin{cases} 1 & i = \ell, \\ 0 & i \neq \ell, \end{cases} \tag{3.2}$$

But it is difficult to get the correct data, which the correlation factor should take in order to guarantee good results. So that k^{th} theoretical damage scenario seems to be the appropriate model when:

$$R(x_i, y_{k\ell}) = \begin{cases} > 0.9 & i = \ell, \\ < 0.1 & i \neq \ell. \end{cases} \tag{3.3}$$

Denote $R_k = \{r\}_{i,\ell=1,n} = \{R(x_i, y_{k\ell})\}_{i,\ell=1,n}$ is the correlation matrix between x_i and $y_{k\ell}$ for k^{th} theoretical damage case, so k^{th} theoretical damage scenario is the appropriate model if

$$\begin{aligned} \{r\}_{i,j} &> 0.9 \quad i = \ell, \\ \{r\}_{i,j} &< 0.1 \quad i \neq \ell. \end{aligned}$$

According to the restriction of the detection criteria, the value 0.9 and 0.1 can be changed. Using expression (3.1) mode shapes are not required to normalize because it is normalized automatically. When all the damage scenarios do not satisfy (3.3), it means that the damage library does not have a theoretical model corresponding with the experimental structure. The new model need to be determined and add to damage library to make the library more complete.

4. The simulated example

The experimental and theoretical data used on this study come from tests performed on the I-40 Bridge over the Rio Grande in New Mexico. Figure 1 shown an elevation view of the portion of the bridge and its cross-section geometry.

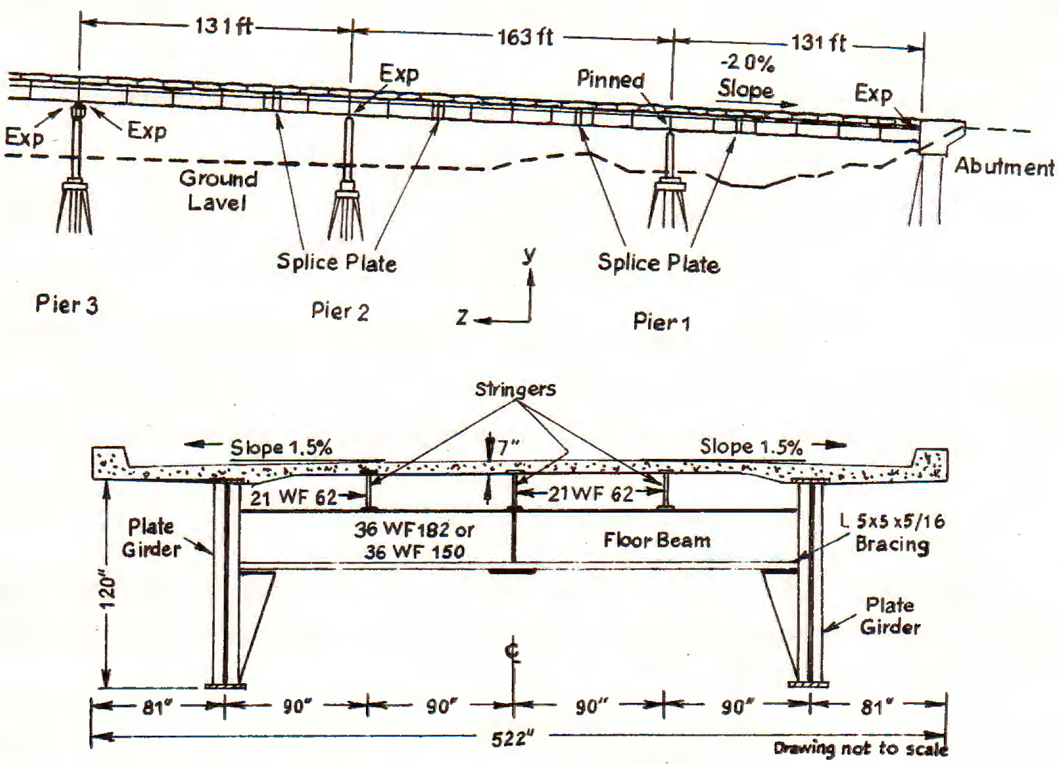


Fig. 1. The I-40 bridge

In the case study, a damage library is given from measured data, then theoretical data is used to detect the mathematicsl model of the bridge.

4.1. Damage Description

In the damage library, there are 5 cases of measured modal data:

+ Un damage.

+ Damage: Damage was introduced by making various torch cuts in the web and flange of the north girder. It contains:

- E-1: 2 foot long, 3/8 inch wide cut through web centered at midheight of the web.

- E-2: First cut was continued to the bottom of the web.

- E-3: The flange was then cut halfway in from either side directly below the cut in the web.

- E-4: The flange was cut through leaving the top 4^{ft} of the web and the top flange.

These damages and measured mode shapes are given in table 1.

Table 1. Cases for Measured Damage

Natural Frequency in damage cases (Hz)

	Theory	Case 1	Case 2	Case 3	Case 4	Case 5
Mode 1	2.05	2.48	2.52	2.52	2.46	2.30
Mode 2	2.72	2.96	3.00	2.99	2.95	2.84
Mode 3	3.28	3.50	3.57	3.52	3.48	3.49

The first mode shape

	Theory	Case 1	Case 2	Case 3	Case 4	Case 5
N1	0.000548	2.57E-04	3.95E-03	5.56E-03	4.48E-03	8.40E-03
N2	0.039121	6.90E-03	0.218	0.226	0.2	0.101
N3	0.053435	0.01	0.32	0.316	0.295	0.143
N4	0.038016	8.01E-03	0.257	0.246	0.231	0.11
N5	-0.000008	-1.66E-05	-6.08E-03	0.037	3.07E-03	0.013
N6	-0.053636	-0.014	-0.454	-0.425	-0.389	-0.204
N7	-0.07595	-0.023	-0.732	-0.684	-0.636	-0.368
N8	-0.051216	-0.015	-0.491	-0.475	-0.427	-0.251
N9	-0.000081	-4.56E-04	-0.015	-0.016	0.12	-7.19E-03
N10	0.029742	8.62E-03	0.268	0.225	0.25	0.131
N11	0.039107	0.01	0.314	0.306	0.293	0.177
N12	0.026713	7.54E-03	0.23	0.225	0.218	0.118
N13	0.000377	4.23E-04	0.015	9.16E-03	0.014	0.013
S1	0.000546	3.02E-04	0.01	0.013	9.50E-03	0.012
S2	0.039066	7.46E-03	0.263	0.223	0.208	0.204
S3	0.053358	0.01	0.372	0.331	0.297	0.292
S4	0.037979	8.12E-03	0.279	0.26	0.226	0.232

	Theory	Case 1	Case 2	Case 3	Case 4	Case 5
S5	-0.00001	2.01E-04	4.90E-03	7.27E-03	2.68E-03	0.014
S6	-0.053752	-0.014	-0.494	-0.441	-0.405	-0.5
S7	-0.076165	-0.021	-0.741	-0.68	-0.625	-0.964
S8	-0.051378	-0.014	-0.486	-0.454	-0.418	-0.535
S9	-0.000081	-3.67E-04	-0.019	-0.014	-0.013	-0.023
S10	0.029472	7.52E-03	0.278	0.25	0.226	0.252
S11	0.039088	0.01	0.353	0.329	0.313	0.342
S12	0.026691	6.99E-03	0.247	0.221	0.217	0.226
S13	0.000376	4.34E-04	0.013	0.014	0.011	0.011

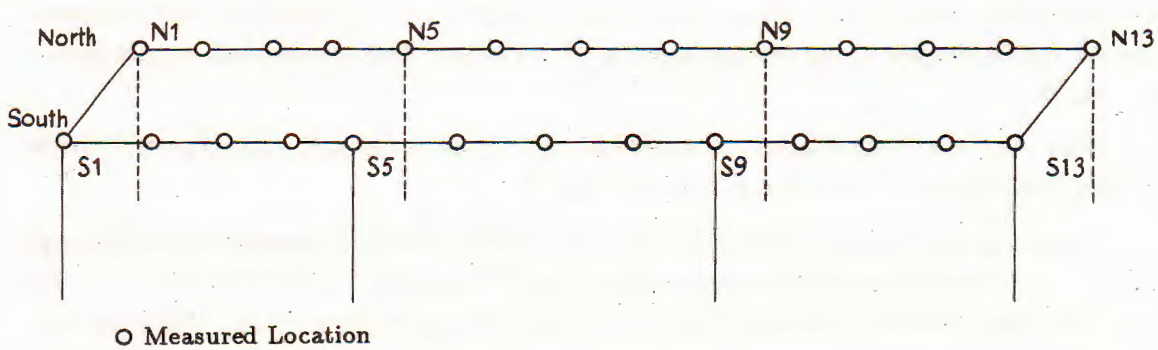
The second mode shape

	Theory	Case 1	Case 2	Case 3	Case 4	Case 5
N1	0.000257	1.73E-04	3.70E-03	3.15E-03	2.90E-03	6.64E-03
N2	0.009626	5.55E-03	0.171	0.194	0.182	0.247
N3	0.021012	8.38E-03	0.258	0.285	0.277	0.365
N4	0.020227	6.98E-03	0.218	0.236	0.23	0.292
N5	-0.000223	-1.37E-04	-3.76E-03	8.37E-03	4.29E-04	2.19E-03
N6	-0.052309	-0.014	-0.433	-0.435	-0.441	-0.495
N7	-0.080272	-0.024	-0.715	-0.711	-0.73	-0.797
N8	-0.056831	-0.016	-0.49	-0.495	-0.498	-0.55
N9	-0.000063	-5.35E-04	-0.016	-0.016	-0.016	-0.015
N10	0.035573	8.87E-03	0.273	0.263	0.277	0.322
N11	0.047852	0.011	0.322	0.307	0.322	0.384
N12	0.033257	8.43E-03	0.246	0.234	0.252	0.297
N13	0.000587	4.72E-04	0.014	0.015	0.017	0.017
S1	-0.000254	-2.93E-04	-6.18E-03	-7.03E-03	-8.38E-03	-4.87E-04
S2	-0.00952	-7.62E-03	-0.222	-0.211	-0.234	-0.104
S3	-0.020799	-0.011	-0.322	-0.313	-0.34	-0.154
S4	-0.020065	-8.42E-03	-0.242	-0.229	-0.252	-0.122
S5	0.000225	1.11E-04	1.73E-03	4.84E-03	1.24E-03	7.41E-04
S6	0.052253	0.016	0.468	0.464	0.48	0.298
S7	0.080227	0.024	0.714	0.711	0.737	0.559
S8	0.056803	0.017	0.49	0.479	0.504	0.323
S9	0.000062	4.27E-04	0.03	0.022	0.021	0.016
S10	-0.03545	-9.97E-03	-0.292	-0.279	-0.26	-0.116
S11	-0.047686	-0.014	-0.39	-0.377	-0.378	-0.169
S12	-0.033145	-9.54E-03	-0.268	-0.258	-0.261	-0.108
S13	-0.000586	-5.61E-04	-0.014	-0.016	-0.021	-6.02E-03

The third mode shape

	Theory	Case 1	Case 2	Case 3	Case 4	Case 5
N1	0.000839	4.72E-04	8.17E-03	6.18E-03	8.38E-03	0.016
N2	0.053185	0.014	0.443	0.393	0.433	0.405
N3	0.062276	0.019	0.597	0.53	0.583	0.543
N4	0.034238	0.013	0.404	0.355	0.392	0.364
N5	0.000534	7.16E-04	0.024	0.017	0.021	0.021
N6	0.010745	4.48E-03	0.151	0.134	0.152	0.135
N7	0.034365	1.04E-03	0.049	0.043	0.051	0.036
N8	0.033865	3.87E-03	0.119	0.105	0.105	0.102
N9	-0.000128	-5.11E-04	-0.016	-0.015	-0.016	0.015
N10	-0.040139	-0.014	-0.435	-0.392	-0.423	-0.399
N11	-0.059494	-0.02	-0.603	-0.558	-0.597	-0.56
N12	-0.043291	-0.016	-0.497	-0.464	-0.501	-0.469
N13	-0.000762	-8.95E-04	-0.019	-0.025	-0.032	-0.029
S1	0.000845	4.78E-04	0.015	1.30E-02	0.019	0.021
S2	0.053301	0.014	0.448	0.384	0.428	0.418
S3	0.0625	0.018	0.581	0.502	0.559	0.543
S4	0.034395	0.011	0.368	0.311	0.349	0.341
S5	0.000537	6.52E-04	0.024	0.019	0.19	0.022
S6	0.010765	4.28E-03	0.125	0.123	0.129	0.13
S7	0.034527	4.64E-04	0.011	0.015	7.65E-03	0.013
S8	0.034052	3.39E-03	0.123	0.088	0.114	0.105
S9	-0.000126	-4.27E-04	-0.013	-9.05E-03	-0.012	-9.99E-03
S10	-0.040346	-0.012	-0.396	-0.343	-0.366	-0.338
S11	-0.059759	-0.019	-0.599	-0.534	-0.588	-0.563
S12	-0.043462	-0.015	-0.463	-0.415	-0.458	-0.435
S13	-0.000764	-9.81E-04	-0.026	-0.027	-0.036	-0.033

Measured Scheme (in plane)



Two cases of detecting appropriated models (undamaged and E-4 case) will be considered in this paper.

4.2. Finite Element modeling of the I-40 bridge

Based on the I-40 bridge data (Fig. 1), we built the finite element model of the bridge superstructure. This model contains a total of 575 nodes, 604 elements. Four node shell elements were chosen to model the girder flange, the web of two girders, the floor beam, the stringer and the bridge decks. Two node beam elements were used to model the cross-bracing. Detailing of the bridge model at the abutment end is shown on Fig. 2.

For the damage type described above, the change of the finite element model in the damage location is shown in Fig. 3

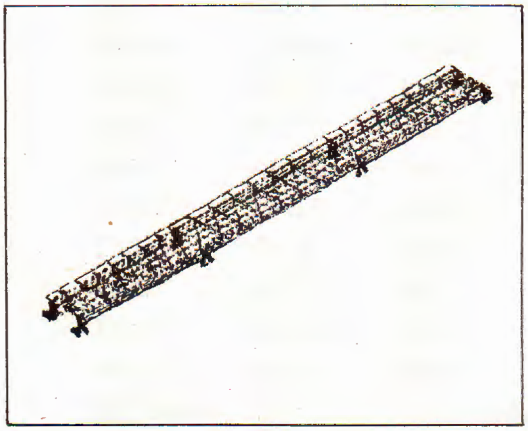


Fig. 2. The finite element model of I-40 bridge

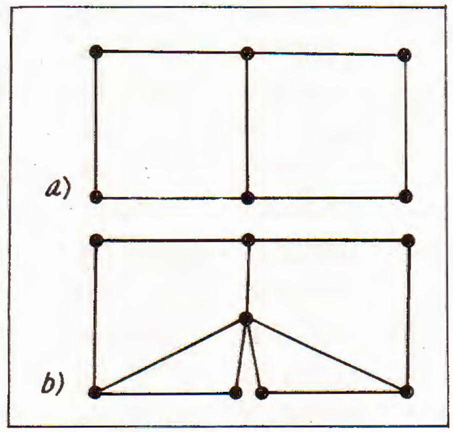


Fig. 3. Finite element modeling before (a) and after (b) damage

4.3. Detecting appropriate models in the damage library

For the convenience of comparison, we develop^d a program (in C language) to display the measured mode shape in graphics mode. Fig. 4 displays three experimental mode shapes using this program and corresponding mode shapes calculated by SAP90.

The results of correlation matrices between theoretical and experimental mode shapes according to (3.1) are given in Table 2.

So, the undamaged model in library is compared with 5 experimental damage cases. The correlation matrices are given in 2^{nd} column of Table 2. We can see that the two first correlation matrices satisfy the condition (3.3), therefore the two first experimental cases are regarded as undamaged. The rest of the cases are considered as damaged. Using the 4^{th} damaged model in the library and comparing it with experimental damaged cases, the results are presented in 3^{rd} column of Table 2. Using condition (3.3), it is easy to find that the 5^{th} experimental damaged case is in accordance with the 4^{th} damaged model in library.

Table 2

The correlation matrices between theoretical and measured mode shape

Case	Undamaged case (in library)	4^{th} damaged cases (in library)
+ Und.	$\begin{pmatrix} 0.974 & 8.04E\text{-}4 & 7.4E\text{-}2 \\ 8.04E\text{-}4 & 0.973 & 2.8E\text{-}4 \\ 7.4E-2 & 2.8E-4 & 0.91 \end{pmatrix}$	$\begin{pmatrix} 0.95 & 2.38E\text{-}2 & 8.77E\text{-}4 \\ 2.38E\text{-}2 & 0.809 & 2.65E\text{-}2 \\ 9.63E\text{-}2 & 2.65E\text{-}2 & 0.84 \end{pmatrix}$
+ E-1	$\begin{pmatrix} 0.951 & 4.46E\text{-}4 & 7.64E\text{-}2 \\ 4.64E\text{-}4 & 0.943 & 1.31E\text{-}5 \\ 7.64E\text{-}2 & 1.31E\text{-}5 & 0.907 \end{pmatrix}$	$\begin{pmatrix} 0.96 & 1.13E\text{-}2 & 1.51E\text{-}3 \\ 1.13E\text{-}2 & 0.807 & 2.5E\text{-}2 \\ 1.51E\text{-}2 & 2.5E\text{-}3 & 0.855 \end{pmatrix}$
+ E-2	$\begin{pmatrix} 0.976 & 2.19E\text{-}5 & 7.55E\text{-}2 \\ 2.19E\text{-}5 & 0.969 & 1.26E\text{-}4 \\ 7.55E\text{-}2 & 1.26E\text{-}4 & 0.899 \end{pmatrix}$	$\begin{pmatrix} 0.958 & 1.62E\text{-}2 & 5.76E\text{-}4 \\ 1.62E\text{-}2 & 0.801 & 2.83E\text{-}2 \\ 5.76E\text{-}2 & 2.83E\text{-}2 & 0.85 \end{pmatrix}$
+ E-3	$\begin{pmatrix} 0.969 & 6.99E\text{-}6 & 8.36E\text{-}2 \\ 6.99E-6 & 0.968 & 1.05E\text{-}4 \\ 8.36E\text{-}2 & 1.05E\text{-}6 & 0.882 \end{pmatrix}$	$\begin{pmatrix} 0.95 & 1.85E\text{-}2 & 2.42E\text{-}3 \\ 1.85E\text{-}2 & 0.809 & 2.37E\text{-}2 \\ 2.42E\text{-}2 & 2.73E\text{-}3 & 0.846 \end{pmatrix}$
+ E-4	$\begin{pmatrix} 0.801 & 0.129 & 9.63E\text{-}2 \\ 0.129 & 0.891 & 5E\text{-}3 \\ 9.64E\text{-}2 & 5E\text{-}3 & 0.902 \end{pmatrix}$	$\begin{pmatrix} 0.945 & 5.09E\text{-}2 & 2.31E\text{-}2 \\ 5.09E\text{-}2 & 0.91 & 2.48E\text{-}3 \\ 2.31E\text{-}2 & 2.48E\text{-}3 & 0.907 \end{pmatrix}$

5. Conclusion

Formula (3.1) and condition (3.3) can be used to detect the appropriate model of the structure in the damage library based on the correlative comparison of theoretical and measured mode shapes. Also, this technique can be used to detect the damage location in the structure. When the actual damage does not correspond with one of the modeled scenarios, model updating should be based on other inspection methods (for example, visual and non-destructive methods), and that model can be added to the damage library. If there are some models in the damage library satisfying condition (3.3), it is required to use other methods for support.

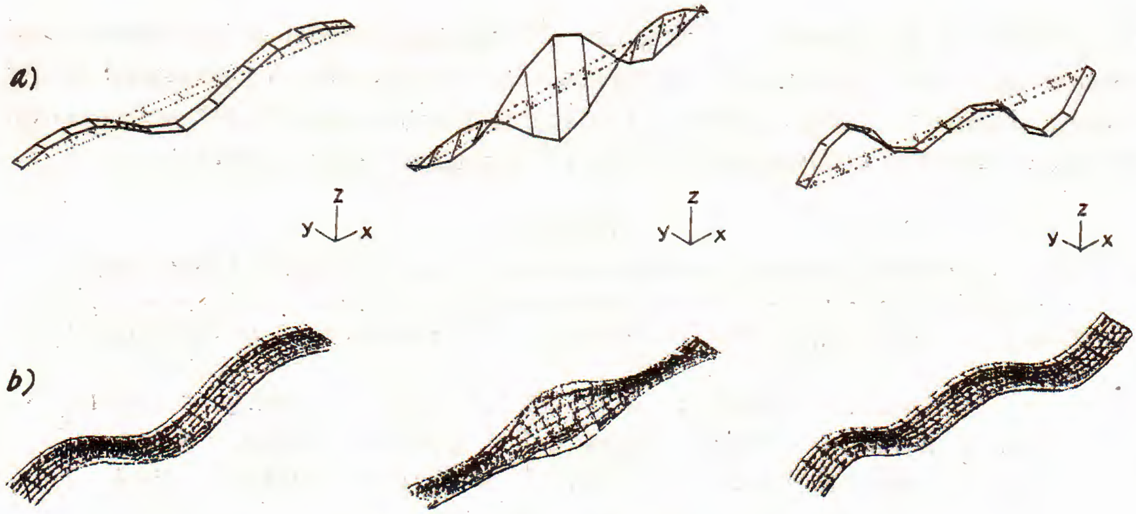


Fig. 4. The experimental and theoretical mode shapes

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SỬ DỤNG CÁC ĐẶC TRƯNG DAO ĐỘNG ĐỂ CẬP NHẬT MÔ HÌNH KẾT CẤU

Việc nghiên cứu cập nhật mô hình tính toán của kết cấu dựa trên các đặc trưng dao động của nó đã được nhiều tác giả nghiên cứu. Một trong các phương pháp được sử dụng là sử dụng các đặc trưng dao động để tìm kiếm mô hình kết cấu phù hợp trong thư viện các hư hỏng đã được tính toán trước. Bài này đề xuất thuật toán tìm kiếm mô hình hư hỏng trong thư viện hư hỏng dựa trên cơ sở phát triển phương pháp tiêu chuẩn bền vững dao động (Modal Assurance Criteria) do Ewins D. J. đưa ra. Các kết quả đo đạc thực nghiệm trong [4] được sử dụng để tính toán minh họa.